tial number of eddies before decaying and that the modification for the interval preceeding the first multiple jump in the criterion, Eq. (7), is important in preventing unacceptable errors. For the most massive particle considered ($d_p = 100\ \mu$m; $\rho_p = 2500$ kg/m^3), $c_0 > \overline{\Delta t/\bar{\tau}}$ and $c = c_0$ for all interactions.

For practical application of the truncation criterion suggested in this technical note, it will certainly be necessary to first validate the criterion for the flow conditions of particular interest, as demonstrated herein for nearly homogeneous turbulent flow. Once established, however, the potential gains in efficiency, through elimination of unnecessary and redundant computational operations, may be significant.

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Cylindrical Bending of Unsymmetric Composite Laminates

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Introduction

EARLY work on unsymmetric laminate analysis was mostly based on the linear classical lamination theory. Recently, Sun and Chin[1,2] found that the linear lamination theory is inadequate for analysis of unsymmetric laminates, and the nonlinear large deflection theory must be used even for problems that are normally considered to be in the small deflection domain. In their studies, however, the transverse shear effect was not considered. This effect is important for resin matrix composite laminates because the interlaminar shear moduli of composite materials are very small as compared with the inplane elastic moduli of reinforced fiber. A recent study by Chen,[3] using a large deflection shear deformation theory on the delamination buckling, postbuckling, and growth behaviors of composite laminates, has demonstrated the importance of transverse shear effects for different delamination sizes.

This Note presents a solution for unsymmetric cross-ply laminates, including both large deflection and transverse shear effects. A pinned-pinned laminated plate subjected to a uniform transverse load is demonstrated. The results from the nonlinear shear deformation theory (NSDT) are also compared with those from the nonlinear classical lamination theory (NCLT),[1,2] the linear shear deformation theory (LSDT), and the linear classical lamination theory (LCLT).

Formulation

Consider an unsymmetric cross-ply laminate subjected to a uniform transverse load. For cylindrical bending, the governing equations are assumed to be independent of the y axis. Based on the large deflection shear deformation formulation,[3] the laminate displacements u and w are expressed by

$$u(x,z) = u^0(x) + z\psi_x(x) \tag{1}$$

$$w(x,z) = w(x) \tag{2}$$

where u^0 is the midplane displacement and ψ_x is the rotation in the xz plane. The equilibrium equations are given by

$$N_{x,x} = 0 \tag{3}$$

$$M_{x,x} - Q_x = 0 \tag{4}$$

$$Q_{x,x} + N_x w_{,xx} + q = 0 \tag{5}$$

where N_x, M_x, and Q_x are membrane force, bending moment, and transverse shear force resultants, respectively, and q is the transverse loading. The constitutive relations for the cross-ply laminates are characterized by

$$N_x = A_{11}(u^0_{,x} + \tfrac{1}{2} w^2_{,x}) + B_{11}\psi_{x,x} \tag{6}$$

$$M_x = B_{11}(u^0_{,x} + \tfrac{1}{2} w^2_{,x}) + D_{11}\psi_{x,x} \tag{7}$$

$$Q_x = kA_{55}(\psi_x + w_{,x}) \tag{8}$$

where A_{11}, D_{11}, B_{11}, and A_{55} are extensional, bending, extension-bending coupling, and transverse shear stiffnesses, respectively, and k is the shear correction factor.

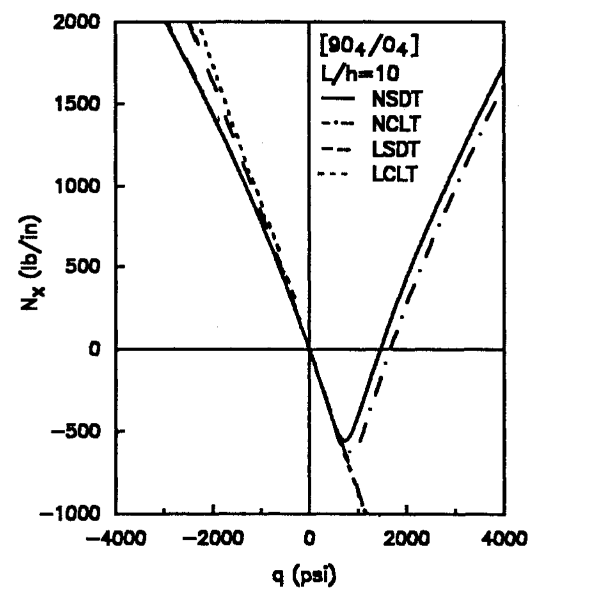

Fig. 1 In-plane force induced by transverse loading based on linear and nonlinear theories.

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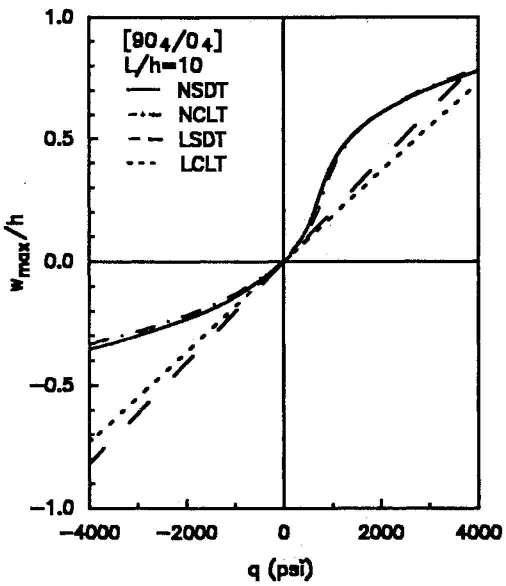

Fig. 2 Maximum deflection due to transverse loading based on linear and nonlinear theories.

From Eq. (3), it is concluded that N_x is a constant. Substitution of Eq. (6) into Eq. (7) yields

$$M_x = \frac{B_{11}}{A_{11}} N_x + D_c \psi_{x,x} \tag{9}$$

where

$$D_c = D_{11} - \frac{B_{11}^2}{A_{11}} \tag{10}$$

By using Eqs. (4), (5), and (7–9), the governing equation for deflection w is derived as

$$w_{,xxxx} - \frac{N_x}{[1 + (N_x/kA_{55})]D_c} w_{,xx} = \frac{q}{[1 + (N_x/kA_{55})]D_c} \tag{11}$$

Note that the formulation for LSDT can be obtained by dropping the nonlinear terms in Eqs. (6–8). The formulations for NCLT and LCLT can be obtained from the NSDT and LSDT formulations, respectively, by neglecting the transverse shear effects, i.e., by assuming the transverse shear stiffness is infinite ($A_{55} = \infty$).

Laminated Plate with Pinned-Pinned Edges

The characteristic of Eq. (11) depends on the coefficient of the second derivative of the transverse deflection where the induced in-plane force N_x determines the form of the displacement solution. For a pinned-pinned laminated plate subjected to a uniform transverse load, it is found that N_x can be positive (tensile), negative (compressive), or zero decided by the sign of the coupling coefficient B_{11}, the sign and magnitude of the transverse load q, and the slenderness ratio of the laminate L/h. These different conditions are discussed individually in the following paragraphs.

Tensile In-Plane Force ($N_x > 0$)

For the case with $N_x > 0$, the governing equation (11) can be expressed as

$$w_{,xxxx} - \alpha^2 w_{,xx} = \frac{q}{[1 + (N_x/kA_{55})]D_c} \tag{12}$$

where

$$\alpha^2 = \frac{N_x}{[1 + (N_x/kA_{55})]D_c} \tag{13}$$

The general solution of Eq. (12) is

$$w(x) = C_2 \cosh \alpha x + C_4 - \frac{q}{2N_x} x^2 \tag{14}$$

For pinned-pined boundary conditions,

$$w(\pm a) = 0 \tag{15}$$

$$M(\pm a) = \frac{B_{11}}{A_{11}} N_x - \left(1 + \frac{N_x}{kA_{55}}\right) D_c w_{,xx}(\pm a) - \frac{D_c}{kA_{55}} q = 0 \tag{16}$$

$$u^0(\pm a) = u^0(0) + \int_0^{\pm a} \left(\frac{N_x}{A_{11}} - \frac{B_{11}}{A_{11}} \psi_{x,x} - \frac{1}{2} w_{,x}^2\right) dx = 0 \tag{17}$$

The three unknowns, N_x, C_2, and C_4, determined from Eqs. (15–17), are given by

$$C_2 = \left(\frac{B_{11}}{A_{11}} + \frac{D_c}{N_x^2} q\right) \Big/ \cosh \alpha a \tag{18}$$

$$C_4 = \frac{qa^2}{2N_x} - \frac{B_{11}}{A_{11}} - \frac{D_c}{N_x^2} q \tag{19}$$

$$\frac{N_x}{A_{11}} + C_2 \frac{B_{11}}{A_{11}} \frac{N_x}{D_c} \frac{\sinh \alpha a}{\alpha a} - \frac{B_{11}}{A_{11}} \frac{q}{N_x} - \frac{1}{4} C_2^2 \alpha^2 \left(\frac{\sinh 2\alpha a}{2\alpha a} - 1\right)$$
$$+ C_2 \frac{q}{N_x}\left(\cosh \alpha a - \frac{\sinh \alpha a}{\alpha a}\right) - \frac{q^2 a^2}{6N_x^2} = 0 \tag{20}$$

Equations (18–20) are three nonlinear algebraic equations coupled together. They can be solved by the Newton-Raphson iterative method. Note that Eq. (20) contains both q and q^2 terms. Therefore, the solutions are different for positive and negative loadings.

Compressive In-Plane Force ($N_x < 0$)

For the case with $P \equiv -N_x > 0$, the governing equation (11) can be expressed as

$$w_{,xxxx} + \beta^2 w_{,xx} = \frac{q}{[1 - (P/kA_{55})]D_c} \tag{21}$$

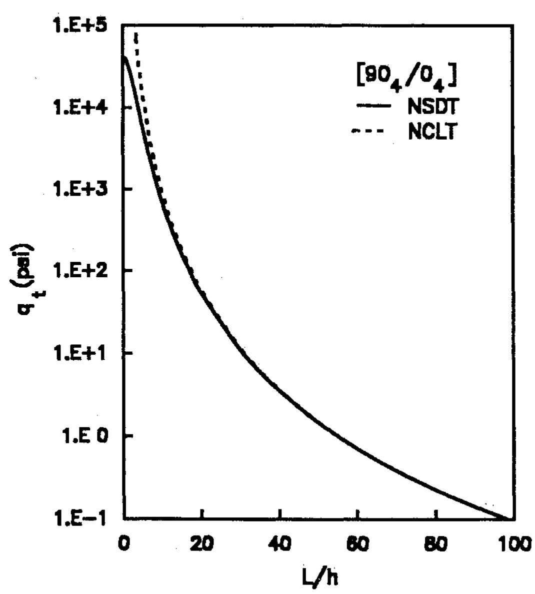

Fig. 3 Variation of transition transverse loading.

where

$$\beta^2 = \frac{P}{[1-(P/kA_{55})]D_c} \tag{22}$$

The general solution of Eq. (21) is

$$w(x) = C_2 \cos \beta x + C_4 + \frac{q}{2P} x^2 \tag{23}$$

The three unknowns, P, C_2, and C_4, can be determined from the boundary conditions, Eqs. (15–17), in a similar way as was the case of $N_x > 0$.

Transition State with Vanishing In-Plane Force ($N_x = 0$)

The state corresponding to $N_x = 0$ is referred to as the "transition state," which implies that the induced in-plane force changes its sign when this state is passed. For this case, the governing equation (11) with a uniform transverse load q_t can be expressed as

$$w_{,xxxx} = \frac{q_t}{D_c} \tag{24}$$

The general solution of Eq. (24) is

$$w = C_2 x^2 + C_4 + \frac{q_t}{24D_c} x^4 \tag{25}$$

The three unknowns, C_2, C_4, and q_t, are determined from the boundary conditions Eqs. (15–17). The resulting transverse deflection w and transverse load q_t are

$$w = \frac{q_t}{24D_c}(5a^2 - x^2)(a^2 - x^2) + \frac{q_t}{2kA_{55}}(a^2 - x^2) \tag{26}$$

$$q_t = -\frac{B_{11}}{A_{11}}\left[\frac{17a^4}{210D_c} + \frac{2a^2}{5kA_{55}} + \frac{D_c}{2(kA_{55})^2}\right]^{-1} \tag{27}$$

It is noted from Eq. (27) that q_t must be in opposite sign as the coupling stiffness B_{11}. Therefore, a transition state can only exist when B_{11} and q are in opposite signs.

Results

Numerical solutions are obtained based on the following graphite/epoxy composite properties: $E_1 = 20 \times 10^6$ psi, $E_2 = 1.4 \times 10^6$ psi, $G_{12} = G_{13} = 0.8 \times 10^6$ psi, $G_{23} = 0.6 \times 10^6$ psi, $\nu_{12} = 0.3$. According to Ref. 4, the shear correction factor k is taken as $\pi^2/12$. The ply thickness is 0.005 in. For the laminate $[90_4/0_4]$, Figs. 1 and 2 show the comparison between the nonlinear solutions and the linear solutions. The induced in-plane force and maximum deflection predicted by the linear theories (LSDT and LCLT) are proportional to the transverse load. However, the nonlinear theories (NSDT and NCLT) give quite different results. As shown in Fig. 1, negative q induces positive in-plane force N_x with a negative coupling stiffness B_{11}. As discussed earlier, a transition state exists only when B_{11} and q are in the opposite signs. As a result, positive q will induce a compressive in-plane force initially, which turns into tension once q passes the transition transverse load q_t.

Figure 2 shows that the maximum deflections predicted by nonlinear theories are also significantly different from those predicted by linear theories. In addition, the results from NSDT and NCLT also show that the deflection with positive q is larger than that with negative q. This is attributed to the compressive in-plane force initially induced by positive q, which tends to aggravate the transverse deflection. Figure 2 also shows that the transverse shear effects are less significant from the nonlinear theories for the present case.

Theoretically, the transition state exists for every unsymmetric laminate as long as the transverse load q and the laminate coupling stiffness B_{11} have opposite signs. Figure 3 shows the variation of the transition transverse load q_t, obtained from Eq. (27), for laminates with slenderness ratios L/h up to 100. For states above this curve, the induced in-plane forces are in tension. For those below this curve, the induced in-plane forces are in compression. Note that q_t varies drastically with the slenderness ratio. For a thin laminate with rather large slenderness ratio, q_t becomes negligibly small. Since the transverse shear effect is more important for thick plates, it is noted in Fig. 3 that the transverse shear effect on q_t becomes increasingly more pronounced as L/h is less than 20.

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Matrix Transformation Method for Updating Dynamic Model

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Introduction

MANY systematic methods[1-13] have been developed in recent years for updating analytical models to predict modal test data more closely. The methods in Refs. 1–7, referred to as matrix-type procedures here, correct the whole mass and stiffness matrices. Correspondingly, the methods in Refs. 8–13, element-type procedures, modify only some nonzero elements of mass and stiffness matrices. In the former procedures, the connectivity of the original analytical model is not preserved, causing the addition of unwanted load paths. In the latter, particularly in Refs. 9 and 11, the stiffness matrix may be identified exactly in certain cases even when some of the test modes are not known.

The purpose of the work presented in this Note is similar to that of Ref. 8. This Note proposes a matrix transform method (MTM), in which the derivations are much simpler than those in Refs. 1–6, which use Lagrange multipliers. In the present MTM, transform matrices for the correction of dynamic models are created. The effect of the transformation is opposite to that of the objective function in the Lagrange multiplier method (LMM); the former is to reduce the number of unknown parameters of the governing equations and the latter is to increase the number of equations. Not only can MTM reproduce the formulas in Refs. 1–6, but it can also derive some new formulations that are difficult to be formed by LMM.

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